7.1-7.2: Sample Spaces, Events and Relative Frequencies

Definition 1. An experiment is some occurence which has a result, or outcome, that is uncertain before the experiment takes place. The set of all possible outcomes is called the sample space for the experiment.

Example 1.

(a) Flipping a (fair or otherwise) coin is an experiement which has two outcomes making up the sample space $S = \{H, T\}$.

(b) Rolling a die is an experiement which has six outcomes making up the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

(c) Drawing a 5-card poker hand from a standard deck of 52 cards is an experiement which has $C(52, 5)$ outcomes making up the sample space. An example of an element in the sample space is

Royal Flush={(A, Spades), (K, Spades), (Q, Spades), (J, Spades), (10, Spades)}.

(d) Selecting a student in the class is an experiment which has as many outcomes as students in the class. The sample space is the set of students in the class.

(e) Rolling two dice is an experiment which could have any number of different outcomes depending on what you wish to determine from the experiment. If you suppose the dice are distinguishable and care about the faces of each die individually, there are 36 outcomes. If you suppose the dice are indistinguishable and care about the faces of the dice as a whole, there are 21 outcomes. If you care care about the sum of the two dice rolled, there are 11 outcomes. If you care about the number of dice rolled which are even, there are 3 outcomes.

Definition 2. Given a sample space S, an event E is a subset of S. We will often refer to an outcome in E as a success or favorable outcome and an outcome not in E as a failure or unfavorable outcome. We say that the event E occurs or is successful in a particular experiment if the outcome of that experiment is one of the elements of E.

Question 1. For each of the experiments in Example 1, come up with a non-trivial (not the empty set) event E.

Example 2. Suppose two distinguishable dice are rolled so that the sample space is $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$. Find the subset of S which describes mathematically each event described in words.

(a) E : The sum of the numbers showing is 6.

(b) F : The sum of the numbers showing is 2.

(c) G : The sum of the numbers showing is 1.

Remark 1. Everything that can be done to sets can also be done to events. For instance, sometimes the event we care about can be described in words with compound statements involving "or," "and," or "not." In these instances, the subsets of the sample space would be described by unions, intersections or complements, respectively.

Definition 3. Two events E and F are said to be disjoint or mutually exclusive if $E \cap F = \emptyset$.

Example 3.

(a) When rolling a die, the events E : the roll is even and F : the roll is a 3 are mutually exclusive.

(b) When flipping a coin three times, the events $E = \{HHH, TTT\}$ and $F = \{HTT, THT, TTH\}$ are mutually exclusive.

(c) For any experiment and event $E \subset S$. The events E and E' are mutually exclusive.

(d) When drawing a poker hand, the events E : a flush is drawn and F : a straight is drawn are not mutually exclusive because the event $E \cap F$: a straight flush is drawn is non-empty.

Definition 4. When an experiment is performed a number of times, the relative frequency or estimated probability of an event E is the fraction of times that the event E occurs. If the experiment is performed N times and the event E occurs $fr(E)$ times, then the relative frequency is given by

$$P(E) = \frac{fr(E)}{N}.$$

The number $fr(E)$ is called the frequency of E. N is called the number of trials or the sample size. The collection of the estimated probabilities of all the outcomes is the relative frequency distribution or estimated probability distribution.

Example 4.

(a) **Experiment:** Roll a pair of dice and add the numbers that face up.
Event: The sum is 5.
If the experiment is repeated 100 times and E occurs on 10 of the rolls, then the relative frequency of E is

$$P(E) = \frac{fr(E)}{N} = \frac{10}{100} = 0.10.$$

(b) If 10 rolls of a single die resulted in the outcomes 2, 1, 4, 4, 5, 6, 1, 2, 2, 1, then the associated estimated probability distribution is shown on the following table:

Outcome	1	2	3	4	5	6
Rel. Frequency	.3	.3	0	.2	.1	.1

Question 2. In a survey of 250 hybrid vehicles sold in the US, 125 were Toyota Prii, 30 were Honda Civics, 20 were Toyato Camrys, 15 were Ford Escapes, and the rest were other makes. What is the relative frequency that a hybrid vehicle sold in the US is not a Toyota Camry?

Some Properties of the Relative Frequency Distributions.

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a sample space and let $P(s_i)$ be the relative frequency of the event $\{s_i\}$. Then

1. $0 \leq P(s_i) \leq 1$
2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$
3. If $E = \{e_1, e_2, \ldots, e_r\}$, then $P(E) = P(e_1) + P(e_2) + \cdots + P(e_r)$.

Question 3. Suppose a fair coin is tossed and let E be the event that heads is face up. If the coin is tossed N times, we have seen that the relative frequency (or estimated probability) is given by

$$P(E) = \frac{fr(E)}{N}.$$

What do you expect to happen to $P(E)$ as the number of trials, N, tends to infinity? In other words, how often would you expect to see heads if we did more and more trials?